Event-based Simulation Model for Quantum Optics Experiments

H. De Raedt[*] and K. Michielsen[†]

[*]*Department of Applied Physics, Zernike Institute for Advanced Materials,*
University of Groningen, Nijenborgh 4, NL-9747 AG Groningen, The Netherlands
[†]*Institute for Advanced Simulation, Jülich Supercomputing Centre,*
Research Centre Jülich, D-52425 Juelich, Germany

Abstract. We present a corpuscular simulation model of optical phenomena that does not require the knowledge of the solution of a wave equation of the whole system and reproduces the results of Maxwell's theory by generating detection events one-by-one. The event-based corpuscular model gives a unified description of multiple-beam fringes of a plane parallel plate and single-photon Mach-Zehnder interferometer, Wheeler's delayed choice, photon tunneling, quantum eraser, two-beam interference, double-slit, Einstein-Podolsky-Rosen-Bohm and Hanbury Brown-Twiss experiments. We also discuss the possibility to refute our corpuscular model.

INTRODUCTION

The corpuscular theory of light by Newton and his followers was abandoned in favor of extensions of Huygens' wave theory, culminating in Maxwell's theory of electrodynamics [1–3]. Maxwell's theory is extremely powerful as it applies to all electrodynamic phenomena that find practical, real-life applications. With Einstein's explanation of the photoelectric effect in terms of photons, that is in terms of indivisible quanta of light, the idea of a corpuscular description of light revived. Einstein's hypothesis of light quanta gave birth to the quantum description of light. As the photoelectric effect can be explained by treating the electromagnetic field without assuming the existence of photons [3], the photoelectric effect itself does not indicate that light consists of indivisible particles. However, the experiments by Grangier *et al.* [4] reports clear and direct evidence for the indivisibility of the single photons [3].

A key feature of the experiments by Grangier *et al.* [4] is the use of the three-level cascade photon emission of the calcium atom. It is observed that this atom may emit two photons in two spatially well-separated directions, allowing for the cascade emission to be detected using a time-coincidence technique [5].

In Fig. 1 we show a diagram of the experiment reported in Ref.[4]. One of two light beams produced by the cascade process is directed to detector D. The other beam is sent through a 50-50 beam splitter to detectors D_0 and D_1. Time-coincidence logic is used to establish the emission of the photons by the three level cascade process: Only if detector D and D_0, D_1 or both fire, a cascade emission event occurred.

A first series of experiments corresponds to the situation in which the right-most
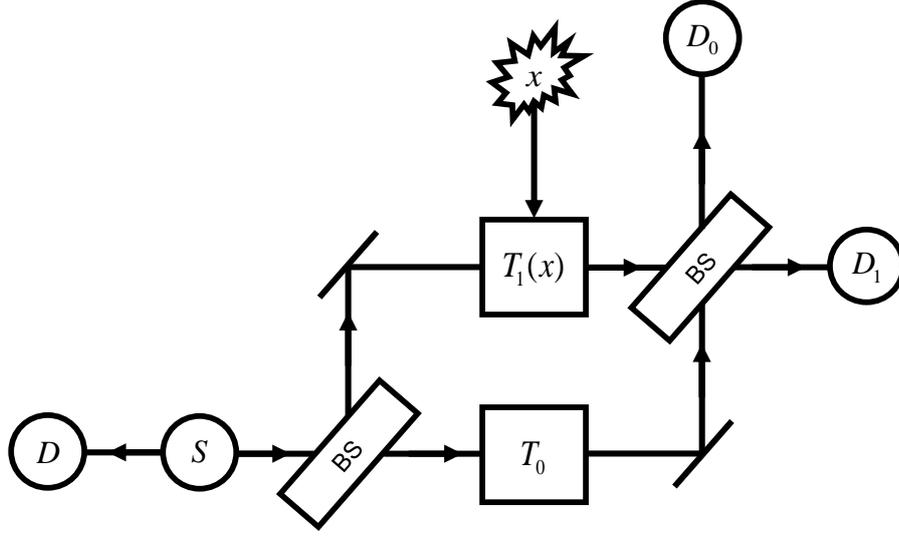
FIGURE 1. Schematic diagram of the experiment performed by Grangier *et al.* [4]. S: Light source; BS: 50-50 beam splitter; T_0: Fixed time-of-flight; $T_1(x)$: Variable time-of-flight controlled by the external variable x; D_0, D_1: Detectors. For clarity, the coincidence logic has been omitted.

beam splitter has been removed. Then, the absence of a coincidence between the firing of detectors D_0 and D_1 provides unambiguous evidence that the photon created in the cascade and passing through the left-most beam splitter behaves as one indivisible entity. The analysis of the experimental data strongly supports the hypothesis that the photons created by the cascade process in the calcium atom are to be regarded as indivisible [3].

Having established the indivisible, corpuscular nature of single photons, a second series of experiments was performed with the right-most beam splitter in place. Then, Grangier *et al.* [4] observe that after collecting many photons one-by-one, the detection counts can be fitted to the prediction of Maxwell's theory for a Mach-Zehnder interferometer (MZI) experiment with a fixed value of x, illuminated by a coherent monochromatic light source S with angular frequency ω, namely [1]

$$I_0 = \sin^2 \frac{\omega(T_0 - T_1(x))}{2} = \sin^2 \frac{\phi_0 - \phi_1(x)}{2}, \tag{1}$$

$$I_1 = \cos^2 \frac{\omega(T_0 - T_1(x))}{2} = \cos^2 \frac{\phi_0 - \phi_1(x)}{2}, \tag{2}$$

where I_0 and I_1 are the normalized intensities recorded by the detectors D_0 and D_1 and $\phi_0 = \omega T_0$ and $\phi_1(x) = \omega T_1(x)$. Equations (1) and (2) show that the signal on the detectors is modulated by the difference between the time-of-flights T_0 and $T_1(x)$ in the lower and upper arm of the interferometer, respectively, or in other words by the phase difference $\phi_0 - \phi_1(x)$. In other words, Grangier *et al.* observe the same result as if the source would have emitted a wave. Therefore, they have demonstrated that the indivisible, corpuscular objects called photons build up an interference pattern one-by-one, just as in the experiment with single-electrons [6–8], for instance.

THE FACTS

Ignoring experimental difficulties such as detection efficiency, instabilities in the MZI an so on, the idealized experiments of Grangier *et al.* (and many others of similar nature) yield the following facts:

1. For every click on detector D either detector D_0 or (exclusive) D_1 click. Each click is regarded as the detection of a single object, called photon. Evidence of indivisibility of this object (defined by the mutually exclusive clicking of D_0 and D_1) establishes the particle character of the photon [3].

2. Many detection events build a histogram that fits well to the intensities obtained by Maxwell's theory implying that the collection of many photons behaves as if it has features that are similar to those of waves.

The conjunction of these two facts cannot be explained within Maxwell's theory or quantum theory. This is most obvious in the case of Maxwell's theory which does not pretend to describe particles. Imagining a single photon to be a spatially localized excitation of a wave field, such a wave packet would, according to wave theory, be divided into two wave packets by a beam splitter. This is not what is observed in the experiments of Grangier *et al.* [4].

Particle-wave duality, a concept of quantum theory which attributes to photons the properties of both wave and particle depending upon the circumstances, does not help to explain the facts either. The experiments [4] show that each individual photon behaves as a particle, not as a wave. In these experiments (and in many others that use single-photon sources), it is clear that one particular photon never interferes with itself nor with other ones; the wave functions that are used in the wave mechanical theory interfere if they interact with material only [9]. It is only in the mathematical, statistical description of many detected photons that (probability) waves interfere [10].

As a last resort to explain the facts using concepts of quantum theory the idea of wavefunction collapse [11] is often used. According to this idea, the wavefunction materializes into a particle during the act of measurement. The mechanism that gives rise to this collapse has remained elusive for 78 years after its conception. Therefore, with the present state of understanding, invoking the wavefunction collapse to explain an experimental observation is no different from invoking magic. Fortunately, this stroke of magic never enters a quantum theoretical calculation of the statistical averages and is, for any practical purpose, superfluous [12]. It only serves to cultivate the belief that quantum theory has something meaningful to say about individual events. However, quantum theory proper only provides a recipe to compute the frequencies (averages) for observing events: It does not describe individual events themselves [13]. Of course, resorting to magic is undesirable from a scientific viewpoint and, as shown in the present paper, also unnecessary to explain the facts.

Logically speaking, there are three options to reconcile the facts with the present state of knowledge:

1. One assumes that quantum theory provides the appropriate set of rules that correctly predicts the probability distribution to observe individual events but one refrains from trying to explain the observation of these individual events themselves. This

"shut up and calculate expectation values" approach is, by construction, free of logical inconsistencies. It has proven to describe many different phenomena very well but does not offer an explanation for, not even any insight about, the process that actually produces the observed events.

2. One simply postulates that it is fundamentally impossible to give an explanation that goes beyond the description in terms of probability distributions to observe events. This is the prevailing position of contemporary quantum physics excluding, by postulate, the possibility that a cause-and-effect explanation may be found.

3. One searches for a logically consistent explanation of the experimental facts, the observed detection events, that does not rely on the knowledge of the probability distribution to observe these events.

In this paper, we explore the last option and demonstrate that it is a viable one. Evidently, persuing this option requires modeling on a level that is not accessible to quantum theory. Our successful demonstration also suggests that it may be time to abandon the second option and instead search for a common-sense, cause-and-effect explanation of the observed facts, as is done in all other fields of science.

The approach presented in this paper gives a cause-and-effect description for every step of the process, starting with the emission and ending with the detection of the photon. By construction, it satisfies Einstein's criterion of local causality. Although not essential, an appealing feature of the approach is that it allows for a realistic interpretation of the variables that appear in the simulation model. Although it is humanly impossible to demonstrate that our approach works for every possible quantum optics experiments that has been and may be conceived in the far future, the fact that the quantum optics experiments such as the Mach-Zehnder interferometer, two beam interference/double slit, Wheeler's delayed choice, quantum eraser and photon tunneling, Einstein-Podolsky-Rosen-Bohm (EPRB) and Hanbury Brown-Twiss (HBT) experiments [14–28], have all been successfully modelled by our approach indicates that our success with this cause-and-effect modelling may be more than an accident. To keep the length of this paper within limits, in the remainder of this paper we limit ourselves to a discussion of the MZI experiment. The key point is to show that quantum optics experiments which are performed in the single-photon regime can be explained entirely

- with an event-based corpuscular model,
- without first solving a wave equation.

The event-based corpuscular model that we discuss in this paper can easily be made universal in that it can, without modification, be used to explain why photons build up interference patterns, why they can exhibit correlations that cannot be explained within Maxwell's theory and so on [28].

COMPUTATIONAL POINT OF VIEW

In this paper we give a partially (partially in the sense that it is impossible to simulate "every" experiment the yet limited time frame) affirmative answer to the fundamental question "Is it possible to simulate, event-by-event, the phenomena observed in real

experiments and reproduce the same statistical answers of experiments and quantum (Maxwell's) theory without any knowledge of the probability (intensity) distributions?". Our reasoning can be summarized as follows:

1. We note that events recorded in experiments can be represented as a string of bits.
2. We ask ourselves: "Can we construct an algorithm (computer program) that generates such strings and their dependence on known parameters with frequencies that agree with quantum theory?"
3. We show that the answer is yes by inventing simple, very short algorithms that specify the relations between cause and effect and that reproduce the facts.

Once we have found a simple set of rules (embodied in the algorithm) that can explain the facts, we may say that we have reached a deeper understanding (although not necessarily the only one) of the process that gave rise to the facts. It is essential to recognize that no such level of understanding can be reached as long as one sticks to a description that involves random variables, simply because by definition, it is supposed to be unknown how the values of these random variables are realized. As is well-known, the values of random variables cannot be generated by an algorithm of finite length and it is obviously impossible to "derive" an algorithm of finite length starting from an algorithm that is unknown.

To simulate experiments such as the one of Grangier *et al.* [4] on a computer, we have two options:

1. First solve the Schrödinger equation (numerically or analytically if possible). Then use pseudo-random numbers to generate events according to the probability distribution obtained from this solution of the Schrödinger equation.
2. Do not make any reference to quantum theory but instead invent an Einstein local, causal process that generates events such that the frequency distribution of many events agrees with the one found in experiment (and with the solution of the Schrödinger equation).

In this paper, we do not consider the first option which, from a conceptual point of view is trivial. Indeed, as it first requires the solution of the Schrödinger equation (the Bohm trajectory description also requires knowledge of this solution), hence makes use of quantum theory, it has nothing meaningful to say about the mechanism that generates the events [13].

Therefore, we focus on the second option, namely the challenge to find algorithms that simulate, event-by-event, the experimental observations that, for instance, interference patterns appear only after a considerable number of individual events have been recorded by the detector [8], without making any reference to concepts of quantum theory. To head off possible misunderstandings, we are not concerned with an interpretation or an extension of quantum theory nor does the success of our approach affect the validity and applicability of quantum theory as such.

In the popular magazines, it is often stated that it is impossible to simulate quantum phenomena by classical processes. Such statements are thought to be a direct consequence of Bell's theorem [29] but are in conflict with other work that has pointed out the irrelevance of Bell's theorem [30–55]. The latter conclusion is supported by several
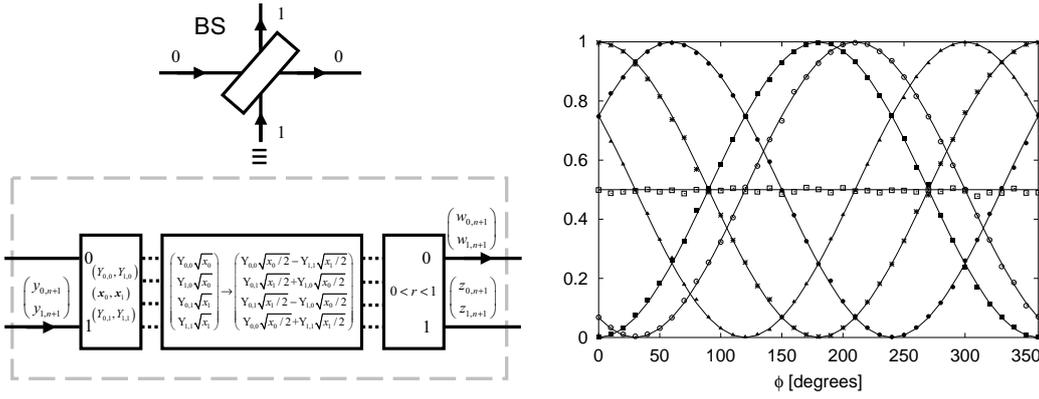
FIGURE 2. Left: Diagram of the DLM that performs an event-by-event simulation of a single-photon beam splitter (BS). The solid lines represent the input and output channels of the BS. Dashed lines indicate the flow of data within the BS. Right: Simulation results for the MZI. Input channel 0 receives $(\cos\psi_0, \sin\psi_0)$ with probability one. A uniform random number in the range $[0, 360]$ is used to choose the angle ψ_0. Input channel 1 receives no events. Each data point represents 10000 events. Initially the rotation angle $\phi_0 = 0$ and after each set of 10000 events, ϕ_0 is increased by $10°$. Markers give the simulation results for the normalized intensity on detector D_0 as a function of $\phi = \phi_0 - \phi_1(x)$. Open squares: Results if the right-most beam splitter is removed; Solid squares: $\phi_1(x) = 0$; Open circles: $\phi_1(x) = 30°$; Bullets: $\phi_1(x) = 240°$; Asterisks: $\phi_1(x) = 0$; Solid triangles: Normalized intensity on of detector D_1 for $\phi_1(x) = 300°$. Lines represent the results of wave theory, see Eqs.(1) and (2). Note that the detection efficiency of the detector model employed is 100%.

explicit examples of algorithms that satisfy Einstein's criteria for locality and causality, yet reproduce *exactly* the two-particle correlations of a quantum system in the singlet state, without invoking any concept of quantum theory [18–21, 23, 56]. Bell's no-go theorem is of very limited value: It applies to a marginal class of classical models which are only relevant to EPRB experiments that are performed in the laboratory if the co-incidence window W approaches infinity (on the time scale of the experiment). From the viewpoint of simulating event-based phenomena on a digital computer, Bell's no-go theorem is of no relevance whatsoever.

SIMULATION MODEL

In our simulation approach, every essential component of the laboratory experiment, including the source and the detectors, have a counterpart in the algorithm. The data is analyzed by counting detection events, just as in the laboratory experiment. The simulation model is solely based on experimental facts and trivially satisfies Einstein's criterion of local causality.

The simulation can best be viewed as a message-processing and message-passing process routing messengers (= particles) through a network of units that processes messages. The processing units, called deterministic learning machines (DLMS) [15, 28], play the role of the components of the laboratory experiment and the network represents the complete experimental set-up. We now specify the operation of the basic component, the beam splitter, of an event-by-event simulation model for the MZI.

Figure 2(left) shows the schematic diagram of the algorithm that simulates a beam splitter. We label events by a subscript $n \geq 0$. At the $(n+1)$th event, the DLM receives a message on either input channel 0 or 1, never on both channels simultaneously. Every message consists of a two-dimensional unit vector $\mathbf{y}_{n+1} = (y_{0,n+1}, y_{1,n+1}) = (\cos(\omega\tau), \sin(\omega\tau))$ that encodes the time-of-flight τ of the messenger.

The first stage of the DLM stores the message $\mathbf{y}_{n+1}$ in its internal register $\mathbf{Y}_k$. Here, $k = 0$ (1) if the event occurred on channel 0 (1). The first stage also has an internal two-dimensional vector $\mathbf{x} = (x_0, x_1)$ with the additional constraints that $x_i \geq 0$ for $i = 0, 1$ and that $x_0 + x_1 = 1$. After receiving the $(n+1)$-th event on input channel $k = 0, 1$ the internal vector is updated according to the rule

$$x_{i,n+1} = \alpha x_{i,n} + 1 - \alpha \quad \text{if} \quad i = k \quad , \quad x_{i,n+1} = \alpha x_{i,n} \quad \text{if} \quad i \neq k, \tag{3}$$

where $0 < \alpha < 1$ is a parameter that controls the adaptiveness of the DLM [15, 28]. By construction $x_{i,n+1} \geq 0$ for $i = 0, 1$ and $x_{0,n+1} + x_{1,n+1} = 1$. Hence the update rule Eq. (3) preserves the constraints on the internal vector. Obviously, these constraints are necessary if we want to interpret the $x_{k,n}$ as (an estimate of) the probability for the occurrence of an event of type k.

The second stage of the DLM takes as input the values stored in the registers $\mathbf{Y}_0$, $\mathbf{Y}_1$, $\mathbf{x}$ and transforms this data according to the rule

$$\frac{1}{\sqrt{2}} \begin{pmatrix} Y_{0,0}\sqrt{x_0} - Y_{1,1}\sqrt{x_1} \\ Y_{0,1}\sqrt{x_1} + Y_{1,0}\sqrt{x_0} \\ Y_{0,1}\sqrt{x_1} - Y_{1,0}\sqrt{x_0} \\ Y_{0,0}\sqrt{x_0} + Y_{1,1}\sqrt{x_1} \end{pmatrix} \longleftarrow \begin{pmatrix} Y_{0,0}\sqrt{x_0} \\ Y_{1,0}\sqrt{x_0} \\ Y_{0,1}\sqrt{x_1} \\ Y_{1,1}\sqrt{x_1} \end{pmatrix}, \tag{4}$$

where we have omitted the event label $(n+1)$ to simplify the notation. Note that the second subscript of the $\mathbf{Y}$-register refers to the type of input event.

The third stage of the DLM responds to the input event by sending a message $\mathbf{w}_{n+1} = (Y_{0,0}\sqrt{x_0} - Y_{1,1}\sqrt{x_1}, Y_{0,1}\sqrt{x_1} + Y_{1,0}\sqrt{x_0})/\sqrt{2}$ through output channel 0 if $w_{0,n+1}^2 + w_{1,n+1}^2 > r$ where $0 < r < 1$ is a uniform random number. Otherwise the back-end sends the message $\mathbf{z}_{n+1} = (Y_{0,1}\sqrt{x_1} - Y_{1,0}\sqrt{x_0}, Y_{0,0}\sqrt{x_0} + Y_{1,1}\sqrt{x_1})/\sqrt{2}$ through output channel 1. Finally, for reasons of internal consistency of the simulation method, it is necessary to replace $\mathbf{w}_{n+1}$ by $\mathbf{w}_{n+1}/\|\mathbf{w}_{n+1}\|$ or $\mathbf{z}_{n+1}$ by $\mathbf{z}_{n+1}/\|\mathbf{z}_{n+1}\|$ such that the output message is represented by a unit vector.

It is an almost trivial exercise to perform an event-by-event computer simulation of the MZI experiment using the DLMs as basic building blocks. The results shown in Fig. 2, demonstrate that DLM-networks accurately reproduce the probabilities of quantum theory for these single-photon experiments [14–17, 28].

One may wonder what the DLM-algorithms have to do with the (wave) mechanical models that we are accustomed to in physics. First, one should keep in mind that the approach that we describe in this paper is capable of giving a rational, logically consistent description of event-based phenomena that cannot be incorporated in a wave mechanical theory without adding logically incompatible concepts such as the wave function collapse [13]. Second, the fact that a mechanical system has some kind of memory and is able to learn is not strange at all, in particular not when two or more

physical systems interact. For instance, a pulse of light that impinges on a beam splitter induces a polarization in the active part (usually a thin layer of metal) of the beam splitter [1]. Assuming a linear response (as is usually done in classical electrodynamics), we have $\mathbf{P}(\mathbf{r},t) = \chi(\mathbf{r},t) * \mathbf{E}(r,t)$ where "$*$" is a shorthand for convolution. If the susceptibility $\chi(\mathbf{r},t)$ has a nontrivial time dependence (as in the Lorentz model [1] for instance), the polarization will exhibit "memory" effects and will "learn" from subsequent pulses. DLMs mimic this behavior in the most simple manner, but on an event-by-event basis.

CONCLUSION

The simulation approach that we have discussed provides a logically consistent, cause-and-effect, ontological description of quantum optics phenomena. The salient features of these simulation models are that they

1. generate, event-by-event, the same type of data as recorded in experiment,
2. analyze data according to the procedure used in experiment,
3. satisfy Einstein's criterion of local causality,
4. do not rely on any concept of quantum theory or probability theory,
5. reproduce the averages that we compute from wave or quantum theory.

We may therefore conclude that this computational modeling approach opens new routes to ontological descriptions of event-based phenomena.

An important question is whether our event-based corpuscular approach predicts new phenomena that can be tested experimentally. Elsewhere [28] we show that after a few hundreds of photons have been processed by the algorithm, the frequencies of observations are hardly distinguishable from the intensities expected from Maxwell's theory (note that some of our event-based models of detectors have 100% detection efficiency [28]). Therefore, to discover new phenomena, one has to conceive an experiment that is capable of testing the transient regime of the message-passing system, that is the regime before the message-passing system reaches its stationary state. Elsewhere we have proposed an experiment with a Mach-Zehnder interferometer that might be used for this purpose [57]. We hope that our simulation results will stimulate the design of new time-resolved single-photon experiments to test our corpuscular model for optical phenomena.

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